Kendal's tau

Kendal's tau is a measure of rank correlation and measures the similarity of orderings of data when ranked by each of the quantities.

If $(x_1, y_1), \dots, (x_n, y_n)$ are a set of joint observations from two random variables X and Y then pairs (x_i, y_i) and $\left(x_j, y_j\right)$ $i \neq j$ are said to be 'concordant' if both $x_i > x_j$ and $y_i > y_j$ (or if both $x_i < x_j$ and $y_i < y_j$). They are said to be 'discordant' if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$ (and neither concordant or discordant if if $x_i = x_j$ or $y_i = y_j$, which will not happen if all the $\{x_i\}$ and all the $\{y_i\}$ are unique).

Kendall's τ coefficient is then:

$$\tau = \frac{(number\ of\ concordant\ pairs) - (number\ of\ discordant\ pairs)}{\frac{1}{2}n(n-1)}$$

As there are $\frac{1}{2}n(n-1)$ pairs in total, the coefficient is in the range $-1 \leq \tau \leq 1$.

[N.B. There are various different ways of handling ties]

It is a non-parametric statistic and focuses just on ordering, i.e. on behaviour of the copula, and not the individual marginal distributions. This accords with how copulas are specified. For certain copula families, the parameter that selects between different members of the family has a one-to-one relationship with Kendal's tau (e.g. the Clayton copula). A natural way of empirically selecting between members of such a family is thus to calculate the empirical Kendal tau (i.e. the one derived from the observations) and then to identify the choice of parameter that reproduces this value.

See MnKendalTauCoefficient or MnKendalTauCoefficients for Nematrian web functions that can be used to calculate Kendal's tau for a single pair of series or for multiple pairs simultaneously.