Moments of a binomial loss distribution

Suppose a portfolio has n equally-sized exposures. Each one is independent and has a probability p of creating a unit loss (and a probability $1-p$ of creating a zero loss), with p the same for each exposure, meaning that the portfolio loss, $-X$, is distributed according to a binomial distribution, i.e.:

$$Pr(X=m)=B(m;n,p)=\binom{n}{m}p^m(1-p)^{(n-m)}=\frac{n!}{m!\,(n-m)!}p^m(1-p)^{(n-m)}$$

The mean and the variance of the portfolio loss distribution can be found as follows. We note that:

$$\sum_{m=0}^{n}\frac{n!}{m!\,(n-m)!}p^m(1-p)^{(n-m)}=1$$

The mean of the loss distribution is given by:

$$E(X)=\sum_{m=0}^{n}\frac{n!}{m!\,(n-m)!}p^m(1-p)^{(n-m)}m$$
$$=np\sum_{m=1}^{n}\frac{(n-1)!}{(m-1)!\,\big((n-1)-(m-1)\big)!}p^{(m-1)}(1-p)^{((n-1)-(m-1))}=np.1=np$$

Likewise:

$$E\big(X(X-1)\big)=n(n-1)p^2\sum_{m=2}^{n}\frac{(n-2)!}{(m-2)!\,\big((n-2)-(m-2)\big)!}p^{(m-2)}(1-p)^{((n-2)-(m-2))}$$
$$=n(n-1)p^2$$

The variance of the loss distribution is:

$$E\left(\big(X-E(X)\big)^2\right)=E((X-np)^2)=E(X^2-2npX+n^2p^2)$$
$$=E\big(X(X-1)\big)+E(X)-2npE(X)+n^2p^2$$
$$=n(n-1)p^2+np(1-2np+np)=np(1-p)$$

Thus binomial distribution has mean np and variance $np(1-p)$.

As $n\to\infty$, the Central Limit Theorem CLT implies that the binomial distribution tends to a normal distribution with the same mean and variance, i.e. to $X\sim N\big(np,np(1-p)\big)$ where $N(x)$ is the cumulative normal distribution.