Formulae for prices and Greeks for European (vanilla) calls in a Black-Scholes world

See Black Scholes Greeks for notation.

Payoff, see MnBSCallPayoff

$$Payoff = \max(S - K, 0)$$

Price (value), see MnBSCallPrice

$$Price = V = Se^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)$$

Delta (sensitivity to underlying), see MnBSCallDelta

$$Delta = \frac{\partial V}{\partial S} = e^{-q(T-t)}N(d_1)$$

Gamma (sensitivity of delta to underlying), see MnBSCallGamma

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{e^{-q(T-t)}N'(d_1)}{S\sigma\sqrt{T-t}}$$

Speed (sensitivity of gamma to underlying), see MnBSCallSpeed

$$Speed = \frac{\partial^3 V}{\partial S^3} = -\frac{e^{-q(T-t)}N'(d_1)\left(d_1 + \sigma\sqrt{T-t}\right)}{\sigma^2 S^2 (T-t)}$$

Theta (sensitivity to time), see MnBSCallTheta

$$Theta = \frac{\partial V}{\partial t} = -\frac{\sigma S e^{-q(T-t)}N'(d_1)}{2\sqrt{T-t}} + qSe^{-q(T-t)}N(d_1) - rKe^{-r(T-t)}N(d_2)$$

Charm (sensitivity of delta to time), see MnBSCallCharm

$$Charm = \frac{\partial^2 V}{\partial S \partial t} = qe^{-q(T-t)}N(d_1) + e^{-q(T-t)}N'(d_1)\left(\frac{d_2}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}}\right)$$

Colour (sensitivity of gamma to time), see MnBSCallColour

$$Colour = \frac{\partial^3 V}{\partial S^2 \partial t} = \frac{e^{-q(T-t)}N'(d_1)}{S\sigma\sqrt{T-t}}\left(q + \frac{1 - d_1 d_2}{2(T-t)} + \frac{d_1(r-q)}{\sigma\sqrt{T-t}}\right)$$

Rho(interest) (sensitivity to interest rate), see MnBSCallRhoInterest

$$Rho(Interest) = \frac{\partial V}{\partial r} = K(T-t)e^{-r(T-t)}\,N(d_2)$$

Rho(dividend) (sensitivity to dividend yield), see MnBSCallRhoDividend

$$Rho(Dividend) = \frac{\partial V}{\partial q} = -S(T-t)e^{-q(T-t)}N(d_1)$$

Vega (sensitivity to volatility), see MnBSCallVega}*

$$Vega = \frac{\partial V}{\partial \sigma} = Se^{-q(T-t)}N'(d_1)\sqrt{T-t}$$

Vanna (sensitivity of delta to volatility), see MnBSCallVanna*

$$Vanna = \frac{\partial^2 V}{\partial S \partial \sigma} = -\frac{d_2 e^{-q(T-t)}N'(d_1)}{\sigma}$$

Volga (or Vomma) (sensitivity of vega to volatility), see MnBSCallVolga*

$$Volga = \frac{\partial^2 V}{\partial \sigma^2} = \frac{d_1 d_2 S e^{-q(T-t)}N'(d_1)\sqrt{T-t}}{\sigma}$$

* Greeks like vega, vanna and Volga/vomma that involve partial differentials with respect to σ are in some sense 'invalid' in the context of Black-Scholes, since in its derivation we assume that σ is constant. We might interpret them as applying to a model in which σ was slightly variable but otherwise was close to constant for all S, t etc.. Vega, for example, would then measure the sensitivity to changes in the mean level of σ. For some types of derivatives, e.g. binary puts and calls, it can be difficult to interpret how these particular sensitivities should be understood.