Annualisation Conventions

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In the following pages we describe some of the different ways in which interest rates and discount rates or yield curves can be expressed and provide tools allowing users to re-express rates using one annualisation convention into rates using a different annualisation convention.

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1. Introduction

A statement from a bank or insurer that a deposit will be credited with interest at the rate of 10% per annum may sound unambiguous. In fact it is not. Interest rates and yields (and hence spreads between different interest rates) can be quoted in a variety of ways.

First we note that market interest rate levels will differ according to the term of the deposit. Thus in practice we might find that the interest rate credited on 1 year deposits is 5% per annum, but on 2 year deposits is 6% per annum, say. This dependency on time is called the 'yield curve'. Interest bearing bonds will have a 'running' (or 'coupon') yield, which is the interest per unit nominal received each year. However, as their price may not be at par, attention is more usually focused on their 'redemption yield', i.e. the rate of return needed to equate their current value with the present value of all future payments, both income and redemption proceeds. With equities, which pay uncertain levels of dividends, the 'dividend yield' may refer to the current dividend rate divided by the current price. The 'rental yield' is a corresponding term that is used for property, i.e. real estate, referring here to the rental income that the property might provide. All of the above may be quoted gross or net of tax if relevant (and potentially also gross or net of other types of expenses).

However these various definitions of yield still do not exhaust the range of meanings that can be placed on a quoted interest rate. In particular, the meaning to be ascribed to any particular interest rate depends on the 'annualisation convention' being adopted.

If, say, a 10% per annum interest rate is being quoted with *annual compounding* the statement that the interest rate is 10% per annum means that at the end of one year 100 grows to:

$$100 \times (1 + 10\%) \equiv 100 \times \left(1 + \frac{10}{100}\right) = 100 \times 1.1 = 110$$

To be meaningful we of course also need to specify the currency (or more generally the 'numeraire') in which value is being expressed, e.g. US$, GB£, Euro, Yen, ...

However, when the interest rate is expressed with *semi-annual compounding*, then the statement means that we earn 5% every six months. This means that at the end of one year 100 grows to:

$$100 \times 1.05 \times 1.05 = 110.25$$

When the interest rate is expressed with *quarterly compounding*, then the statement means that we earn 2.5% every 3 months. This means that at the end of one year 100 grows to:

$$100 \; \times 1.025 \times 1.025 \times 1.025 \times 1.025 = 110.38$$

We can generalise these results by supposing that an amount A is invested for t years at an interest rate of r that is compounded m times per annum. The terminal value of the investment is then:

$$A\left(1+\frac{r}{m}\right)^{mt}$$

The limit as $m \to \infty$ is called *continuous compounding*. The terminal value is then:

$$Ae^{rt}$$

Here e^x, also called $\exp(x)$or $\exp x$, is the exponential function, i.e. $e = 2.718282 \ldots$ raised to the power of x. The continuously compounded interest rate (sometimes also called the *force of interest*) equivalent to a 10% annually compounded interest rate is thus:

$$r = \log_e 1.1 \equiv \ln 1.1 = 0.0953$$

The difference between interest rates quoted with different annualisation conventions increases the further the interest rate deviates from 0%, as is illustrated in Table 1. For most practical purposes, continuous compounding can be thought of as equivalent to daily compounding.

2. ‘Interest’ rates vs ‘discount’ rates

In some circumstances, yields are not quoted in the form of *interest rates* but in the form of *discount rates*. Usually, ‘discount rate’ is used synonymously with ‘interest rate’, as a tool for calculating present values of future cash flows. However, in older literature the two terms may be distinguished, with discount rates being the way in which interest rates are traditionally quoted on discount bills. In such usage ‘discount rates’ would be expressed as fractions or percentages of the final terminal amount, rather than as fractions or percentages of the initial investment. So, if we invest 90 for one year and the terminal amount is 100 then the discount rate (with annual compounding), using this terminology, is 10%. More generally, if an amount A is invested for t years at a *discount* rate, using this terminology, of d that is compounded m times per annum then the terminal value of an investment is:

$$\frac{A}{\left(1-\frac{d}{m}\right)^{mt}}$$

Again we may consider the limit, as $m \to \infty$. We find that the continuously compounded discount rate, using this terminology, is the same as the continuously compounded interest rate, since $e^{rt} = 1/e^{-rt}$.

In actuarial texts, the ‘effective’ interest and discount rates (i.e. annually compounded) are sometimes referred to by i and d respectively and the ‘nominal’ interest and discount rates (compounded m times per annum) are sometimes referred to by $i^{(m)}$ and $d^{(m)}$ respectively (in older texts, $d^{(m)}$ is sometimes instead referred to by $j_{(m)}$). The 'force' of interest and the 'force' of discount (i.e. the continuously compounded rates) are the same and are sometimes referred to by δ. We then have the following relationships.

	Effective interest rate i	Nominal rate of interest $i^{(m)}$	Effective rate of discount d	Nominal rate of discount $d^{(m)}$	Force of interest or discount δ
Value of 1 after time t	$(1+i)^t$	$\left(1+\frac{i^{(m)}}{m}\right)^{mt}$	$(1-d)^{-t}$	$\left(1-\frac{d^{(m)}}{m}\right)^{-mt}$	$e^{\delta t}$
Present value of 1 due after time t	$(1+i)^{-t}$	$\left(1+\frac{i^{(m)}}{m}\right)^{-mt}$	$(1-d)^t$	$\left(1-\frac{d^{(m)}}{m}\right)^{mt}$	$e^{-\delta t}$

Annualisation Conventions: IllustrativeTable

See Annualisation Conventions for a further explanation of annualisation conventions. See MnRestateYieldOrDiscount for a Nematrian web service function that allows you to calculate the impact of different annualisation conventions for arbitrary interest rates.

Illustrative impact of different annualisation conventions, where amounts shown in columns 4, 5, 6 and 7 are amounts by which an investment of 1 will grow to after one year for the given interest / discount rate (with relevant annualisation convention):

Type of rate	Annualisation convention	Parameter used to code this quote basis	Amount after 1 year	Amount after 1 year	Amount after 1 year	Amount after 1 year	Rate equivalent to an annually compounded rate of 10% pa
			-5% pa	1% pa	10% pa	30% pa	pa
'interest'	annual	1	95.00	101.00	110.00	130.00	10.00%
'interest'	semi-annual	2	95.06	101.00	110.25	132.25	9.76%
'interest'	quarterly	4	95.09	101.00	110.38	133.55	9.65%
'interest'	monthly	12	95.11	101.00	110.47	134.49	9.57%
'interest'	weekly	52	95.12	101.00	110.51	134.87	9.54%
'interest'	daily	365	95.12	101.01	110.52	134.97	9.53%
'interest' or 'discount	continuous	0	95.12	101.01	110.52	134.99	9.53%
'discount'	daily	-365	95.12	101.01	110.52	135.00	9.53%
'discount'	weekly	-52	95.13	101.01	110.53	135.10	9.52%
'discount'	monthly	-12	95.13	101.01	110.56	135.50	9.49%
'discount'	quarterly	-4	95.15	101.01	110.66	136.59	9.42%
'discount'	semi-annual	-2	95.18	101.01	110.80	138.41	9.31%
'discount'	annual	-1	95.24	101.01	111.11	142.86	9.09%

Annualisation Conventions: Tools

The Nematrian website provides a web service, MnRestateYieldOrDiscount that converts interest or discount rates quoted using one annualisation convention to equivalent rates using a different annualisation convention. With this function, the relevant annualisation convention is defined by a parameter (taking only integral values) that if zero means continuous compounding, if positive ($= m$) means expressed as an interest rate compounded m times per annum and if negative ($= -m$) means expressed as a discount rate compounded m times per annum.

Usually, derivative pricing algorithms concentrate on continuously compounded rates (i.e. 'forces' of interest), since this simplifies the mathematical presentation. For example, if interest rates vary through time then the accumulated value after time T of an investment of A now (i.e. at time $t = 0$), if the continuously compounded interest rate between now and then is $r(t)$ at time t is:

$$Ae^{\int_0^T r(t)dt} = Ae^{r_{avg}T}$$

where:

$$r_{avg} = \int_0^T r(t)dt$$

This corresponds to the natural way in which we would expect to average a quantity through time. If we used other annualisation conventions then the averaging process involves more complicated mathematical formulae.

The impact of different annualisation conventions is shown in Illustratative table of annualisation conventions.

Exact computation of present values of cash flows generally requires not just information about the annualisation convention applicable to the interest rate or discount rate in question, but also precise information about when the cash flows will occur. In real life, the 'expected' date on which these may occur may not be their actual date of payment, e.g. the specified date may not be a business day. Moreover, if, say, coupon payments are specified as annual percentages of nominal amounts but are paid more frequently than annually, say quarterly, then it become necessary to specify how these annual amounts are apportioned between the different quarter dates. These types of conventions are usually called *day count conventions*.