Market Consistency and Weighted Monte Carlo

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- Market consistency
  - What is it and in what areas of finance (insurance, pensions, banking and asset management) is it relevant?
- Economic Scenario Generators and Monte Carlo simulations
- Weighted Monte Carlo techniques

Some of the ideas explored here are covered in Kemp (2009). *Market Consistency: Model Calibration in Imperfect Markets*

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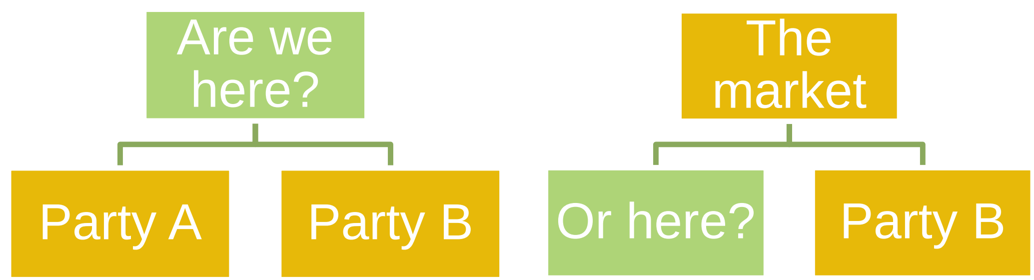
What is ‘market consistency’?

- In e.g. Kemp (2009) the market consistent value of an asset or liability is defined as:
  - Its market value, if it is readily traded on a market at the point in time that the valuation is struck; or
  - A reasoned best estimate of what its market value would have been had such a market then existed, in all other situations
- Closely aligned with the concept of ‘fair value’ used in accountancy
  - I.e. price at which a trade in the asset or liability would take place between a willing buyer and willing seller

What roles do valuations play in finance?

- Ultimately two main roles:
  - To help one party to decide whether now is a good time to buy or sell the asset or liability
  - To apportion value ‘fairly’ between two or more parties
- The two are illustrated by two different job functions within a typical active investment management house



The investment analyst's role

- Tries to identify whether a security is 'cheap' (or 'dear').
  - Compares own assessment of what security is 'truly' worth with market price. Buys or sells depending on sign of difference, if difference large enough
  - Hopes market's view moves closer to his or her view (and quickly enough not to be forced to abandon position in the meantime)
- Deliberately aims to **favour one party** (i.e. the firm's clients) at the expense of some others (i.e. other investors)
  - Outcome is uncertain. Not everyone has the same investment view. Indeed, views need to vary for an active market to exist
  - Clients can't (typically) sue the manager merely for making a wrong investment call, unless some other culpability is established

The unit pricing team's role

- Clients may invest in open-ended pooled funds, e.g. unit trusts, UCITS, mutual funds, Exchange Traded Funds (ETFs), unit-linked life insurance

- Prices are set (usually each day) at which investors can buy and sell units

  - Under modern (UK) regulatory requirements, managers need to avoid creating possible conflicts of interest between themselves and their clients

  - Prices need to be fair, i.e. equitable, **between the parties**

Unit pricing algorithms

- Suppose a firm uses a demonstrably off-market price for unit pricing purposes
- Firm has an asymmetric payoff
  - If unit price is demonstrably too high (or low) then buying clients (or selling clients) suffer and are likely to seek (or need to be given) compensation
  - All roads point to use of market prices, as long as they are readily observable
  - Or some reasonable interpretation of what the market price would be if a ready market existed, in cases where actual market prices are harder to observe

Should valuations target market consistency?

- Valuations come in many forms. Extent to which they need to be “market consistent” varies according purpose

- Different parts of the financial sector may weight the balance differently

market consistency less useful	**Not clear-cut**	**market consistency more useful**
Investment analysis Some valuations for tax purposes, which may focus more on cost or realised gain	Valuations for regulatory capital computations Embedded values Pricing	Unit pricing of mutual funds and equivalents

Banking

- Historically two main ‘books’:
  - Banking book, e.g. loans to individuals or corporates
    - Loans are typically not actively traded
    - Typically priced for regulatory capital purposes at amortised cost
    - Difficult to identify a reliable fair price for such loans
  - Trading book, e.g. trading derivatives, corporate bonds or other instruments
    - More marketable, easier to identify a market price
    - For derivatives, this may involve a model that is designed to bear in mind market observables, i.e. to be ‘market consistent’ (although the term is not used much in the banking sector)

IFRS 9

- A new IAS being introduced, IFRS 9, particularly relevant to banks
  - A conceptually similar subdivision to banking versus trading book still exists, but it is now specified according to the 'business model' adopted for a given portfolio
    - Can be '*hold to collect*' (i.e. aim is to collect contractual cash flows, typically holding instrument to maturity), '*hold to collect and sell*' or '*other*'
    - Only instruments in 'hold to collect' portfolios and whose cash flows are 'Solely Payments of Principal and Interest' can be valued at amortised cost
    - Others need to be fair valued, although sometimes value changes are carried through the other comprehensive income line and sometimes through the P&L
- Regulatory capital calculations include 'filters' so do not necessarily align exactly with values shown in financial statements
- US FASB introducing similar but not identical rules under CECL

Banking

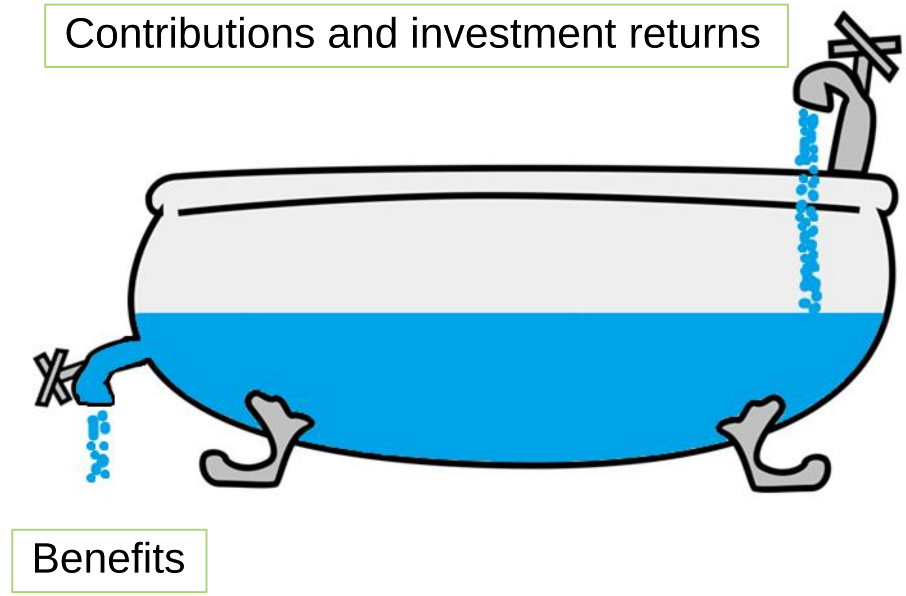
- IFRS 9 is a response to perceived weaknesses in earlier IAS that became evident during the 2007-09 Credit Crisis
  - Previously, instruments would be held at amortised cost until an impairment trigger occurred, which tended to be only when an actual impairment trigger had taken place (e.g. default or extended past due)
  - So perceived to be too **backward-looking**. Evidence suggests banks delayed impairing assets even when it had become clear that they were unlikely to be repaid in full. Inflated their apparent capital position
- G20 leaders and Financial Stability Board asked accounting standards setters to develop something that was more **forward-looking**
  - Under IFRS 9, if credit quality has deteriorated materially then banks are required to allow for lifetime future expected credit losses. Requires enhanced economic modelling

Pension funds

- Two main types of valuation
  - Funding / budgeting valuations
    - Typically, aim is to establish a contribution rate to be paid by sponsor into the fund
    - Usually (in UK) calculation takes some credit for potential future excess returns on risky assets such as equities
    - These excess returns may not materialise (even if they have done so in the past)
  - Discontinuance valuations
    - Typically, aim is to establish whether there are sufficient assets to buy out liabilities with a third party
    - Set bearing in mind market observables (e.g. prices insurers would ask to take on the liabilities)
    - Market prices do not typically take advance credit for risky future returns (instead these are viewed as compensation for the risks being borne)

Budgeting versus matching?

- Is the aim:
  - To smooth the flows and keep the “bath” adequately resourced?
  - To value what is actually in it at any given time?



- See e.g. Cowling, Frankland, Hails, Kemp, Loseby, Orr and Smith (2011). *Developing a framework for the use of discount rates in actuarial work*

- Capacity to be mis-matched often depends on strength of so-called **sponsor covenant**

Insurance

- The EU's Solvency II (introduced 1 Jan 2015) insurance regulatory framework explicitly aims to be market consistent
  - Previous EU regulatory frameworks not harmonised across member states, and valuations of liabilities ('technical provisions') in some cases focused more on a smooth emergence of surplus
- Some areas less market consistent than other areas
  - E.g. Transitional measures
- And identifying what is a market consistent value is not always trivial for some insurance liabilities
  - E.g. very long-term liabilities may extend beyond what is practically observable. Solvency II mandates an "Ultimate Forward Rate"

Solvency II technical provisions

- Article 75:
  - *“Member States shall ensure that, unless otherwise stated, insurance and reinsurance undertakings value assets and liabilities as follows: (a) assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm’s length transaction; (b) liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm’s length transaction.”*
- Insurance liabilities split between best estimate and risk margin. Article 77:
  - “*The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.”*
- Article 79:
  - “*When calculating technical provisions, insurance and reinsurance undertakings shall take account of the value of financial guarantees and any contractual options included in insurance and reinsurance policies.*”

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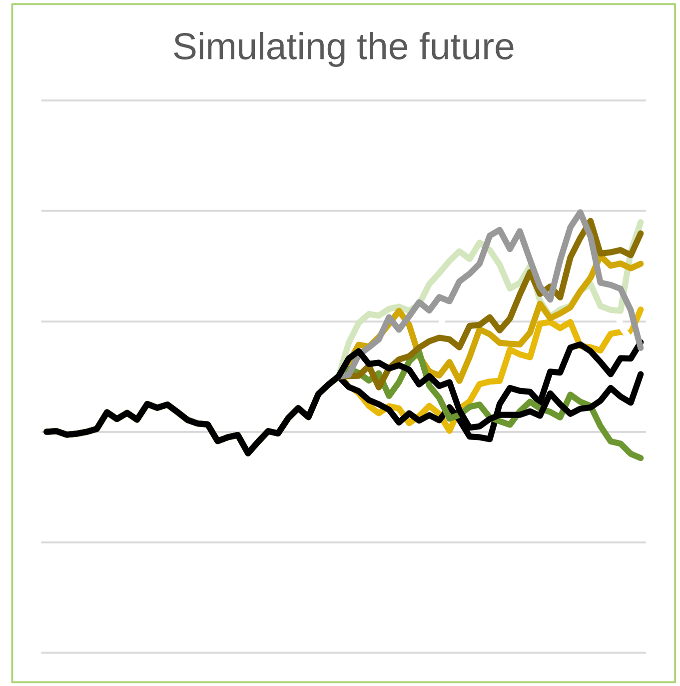
Economic Scenario Generators (ESGs)

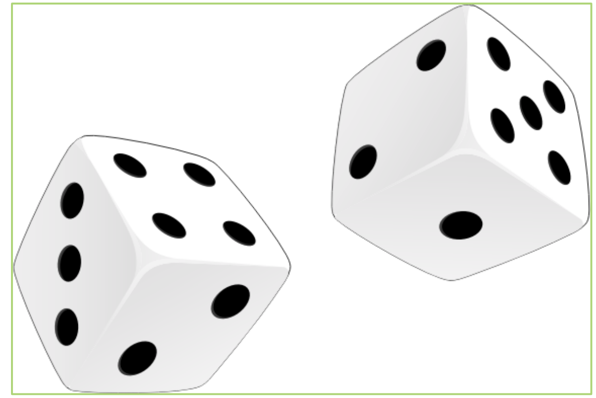
- Solvency II Directive Article 77:
    - *"The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure"*
- Natural actuarial approach, if analytical solution not available, is to estimate probability-weighted cash-flows using simulation techniques
    - I.e. create lots of scenarios about how the future will evolve
- Known in actuarial world as using an **Economic Scenario Generator**



Source: Nematrian

Monte Carlo simulations

- Task is to estimate the probability-weighted cash-flows
- **Monte Carlo:** randomly pick a large number (N) of simulations of how the future might evolve from a suitable distribution
- As $N \rightarrow \infty$ the average present value of the payoff across the simulations tends to the probability-weighted average
- May use variance reduction and other techniques to reduce the N needed to get acceptably close to limiting answer



Real-world versus risk-neutral

- ESGs can be:
  - “Real-world”: allows for risk premia on different assets to assist in selecting investment views to adopt
  - “Risk-neutral”: designed to replicate prices of market observables
- Same subdivision of roles for valuations as covered at start of talk
- For market consistent valuation purposes we need to use **risk-neutral** ESGs or equivalents

Risk-neutral ESGs or equivalents

- Two main approaches, which are mathematically equivalent but present the results somewhat differently:
- Use so-called **deflators**
  - Each simulation has a specified deflator that needs to be applied to it to calculate present values. This approach can be potentially easier to integrate with real-world outputs, if you want your ESG engine to be able to do both simultaneously
- Avoid deflators and discount future cash flows across all simulations at the same **risk-neutral** yield curve
  - More aligned with how derivative pricing is typically done elsewhere in the financial world

Conceptual steps involved with a risk-neutral ESG

- Locate the prices of relevant market observables
  - E.g. prices of zero coupon bonds and of options sensitive to relevant implied volatilities
- Prepare lots of simulations in a way that calibrates to these prices and adheres to other requirements needed for model to be consistent with financial economics principles:
  - E.g. need for series to be martingales (loosely speaking the so-called 1 = 1 requirement, that e.g. the present value of future cash flows arising from an investment of 1 in equities now needs to be 1)
- Project liabilities using these simulations
- Report on results, including demonstrations that model is adequately market consistent

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Weighted Monte Carlo

- Usually Monte Carlo involves equally weighted simulations
  - For e.g. the problem of integrating an unknown function, using equal weights typically minimises variance (hence error) of result
  - Although may use e.g. stratified sampling, relevant sample points then focus on mathematically 'important' part of the problem, e.g. where the integrand is large
- But it is possible to use non-equal weights
- See e.g. Avellaneda et.al. (2001). "*Weighted Monte Carlo: A new technique for calibrating asset-pricing models*" or Kemp (2009)

Conceptual advantages

- More clearly splits up the task between:

  a) Specifying the economic dynamics we think apply to the situation, e.g. correlations between different investment categories

  b) Calibrating the model to market observables (or other externally sourced requirements we want impose on the model)

- Is akin to a Bayesian approach with the prior, i.e. (a), being modified by observations, i.e. (b), to deliver a Bayesian posterior

- Conceptually like (and shares some of the mathematics of) some machine learning techniques, which can also be formulated in Bayesian manner

Mathematical aspects (1)

- Usually we seek weights which are "as similar as possible" to being equally weighted whilst still satisfying the relevant calibration criteria

- As similar as possible usually defined by seeking to minimise relative entropy, i.e. minimise $\sum_i p_i(\log p_i - \log q_i)$ where the q_i are the original weights (here typically $1/N$) and the p_i are the new weights

- Calibrations usually expressed as modelled value needs to match observed value, i.e. $\sum_i p_i PV_i\left(X_j\right) = MV\left(X_j\right)$, say for the j'th calibrating instrument

- The p_i then typically take the form: $p_i = \frac{1}{K}\exp\left(\sum_i z_{ji}\lambda_j\right)$ for some suitable λ_j where the $z_{ji} = PV_i\left(X_j\right)$ are the present values of the payoff on instrument j in the i 'th simulation. K is chosen so that $\sum_i p_i = 1$.

Mathematical aspects (2)

- The λ_j and hence the p_i can then be found using standard optimisation techniques
  - E.g. conjugate gradient, DFP, BFGS, L-BFGS, L-BFGS-B minimisation / optimisation algorithms, see e.g. Press et al. (2007)
  - Algorithms that utilise the (multi-dimensional) gradient of the relative entropy function (plus boundary penalties) are appealing, given computational effort required to calculate the gradients versus an individual function evaluation
  - The problem can also be vectorised and hence parallelised, if suitable parallelisation software and e.g. access to GPUs are available
- Again, similarities with machine learning and related techniques
- **A good route for actuarial students to get ahead of the pack along the 'data science' curve?**

Summary

- Market consistency
  - Widely relevant across financial sector, although term itself is primarily used in insurance and particularly Solvency II
  - Not relevant in all circumstances, particularly when being paid to take views
- Natural actuarial approach, if analytical solution is not readily available, is to estimate probability-weighted cash-flows using simulation, i.e. Monte Carlo, techniques
- ‘Weighted’ Monte Carlo highlights division between model selection and calibration
  - Mathematically similar to some Bayesian techniques used in machine learning

Important Information